1. **Damped Oscillator.** The graph of $f(t) = e^{-t} \sin t$ is an example of a *damped* sine wave. It is used in applications such as the vibrations of a shock absorber.

   (a) Graph f using a calculator or Desmos and explain why this curve is called a damped sine wave.

   (b) Compute $f'(t)$ and use it to determine where f has horizontal tangents.

   (c) Evaluate $\lim_{t \to \infty} f(t)$. What does the result say about the oscillations of this damped sine wave?

2. **Idealistic Vibrations of a Spring.** Suppose an object of mass m is attached to the end of a spring hanging from a ceiling. The mass is at its equilibrium position $y = 0$ when the mass hangs at rest. Suppose you push the mass to a position y_0 and release it. The position of the mass t seconds after release is

$$y = y_0 \cos\left(t\sqrt{\frac{k}{m}}\right)$$

where $k > 0$ is a constant measuring the spring stiffness and $y > 0$ represents a position in an upward direction.

(a) Find $\dfrac{dy}{dt}$ assuming m and k are constant.

(b) How would the velocity be affected if the experiment were repeated with four times the mass on the end of the spring?

(c) How would the velocity be affected if the experiment were repeated with a spring having four times the stiffness? ($K' = 4K$).

(d) Assume y is in meters, t in seconds, m in kilograms, and k has units of kg/s^2. Show that the units of the velocity are consistent.

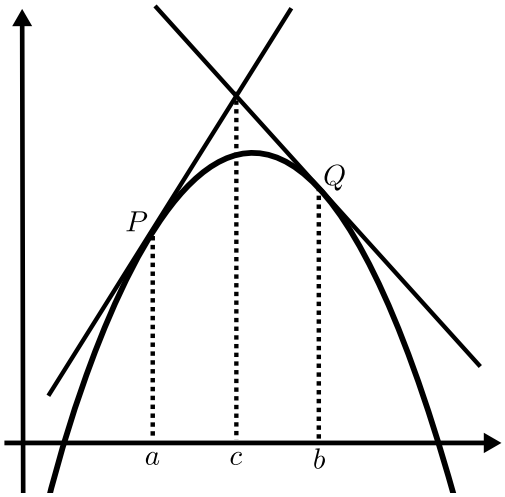
3. **Population Growth.** The population of the state of Washington (in millions) from 2010 ($t = 0$) to 2016 ($t = 6$) is modeled by the function $p(t) = 0.0078t^2 + 0.028t + 6.73$,

   (a) Determine the average population growth rate from 2010 to 2016.

   (b) What was the growth rate in 2011? In 2015?

   (c) Determine all times $0 \leq t \leq 6$ for which the population is increasing.

4. **Means from Tangents.** Let $f(x) = x^2$ and let $P = (a, f(a))$ and $Q = (b, f(b))$ for two real numbers a and b. Show that the abscissa of the intersection of the lines tangent to f at points P and Q is $c = \dfrac{a+b}{2}$, the mean of a and b.



(*cont.*) Use the chain rule to show this same property holds for any parabola (like the random one shown in the above figure). That is, the tangent lines intersect at the mean of a and b.